$$K8 = j\omega\mu_0 \frac{a}{\pi}$$

$$K9 = \frac{j\omega\mu_0\chi_{[11]}a^2b^2}{\pi^2(a^2+b^2)}$$

$$K10 = \frac{j\omega b}{a^2+b^2}\left[\epsilon_0\left(b - h + \frac{b}{2\pi}\sin\frac{2\pi h}{b}\right) + \epsilon_{11}\left(h - \frac{b}{2\pi}\sin\frac{2\pi h}{b}\right)\right]$$

$$K11 = \frac{j\omega a^2}{b(a^2+b^2)}\left[\epsilon_0\left(b - h - \frac{b}{2\pi}\sin\frac{2\pi h}{b}\right) + \epsilon_{22}\left(h + \frac{b}{2\pi}\sin\frac{2\pi h}{b}\right)\right]$$

$$K12 = \frac{j\omega a\sqrt{2}}{\pi(a^2+b^2)^{1/2}}(\epsilon_{22} - \epsilon_0)\sin\frac{\pi h}{b}$$

$$K13 = \frac{a}{b}K10$$

$$K14 = \frac{b}{a}K11$$

$$K15 = \frac{j\omega\chi_{(11)}a^2b}{2\pi^2(a^2+b^2)}\epsilon_{23}\left(1 - \cos\frac{2\pi h}{b}\right)$$

$$K16 = \frac{j\omega}{b}[\epsilon_0(b-h) + \epsilon_{22}h]$$

$$K17 = \frac{j\omega\chi_{(11)}ab\sqrt{2}}{\pi^2(a^2+b^2)^{1/2}}\epsilon_{23}\left(\cos\frac{\pi h}{b} - 1\right)$$

$$K18 = \frac{j\omega\chi_{(11)}ab^2}{2\pi^2(a^2+b^2)}\epsilon_{23}\left(\cos\frac{2\pi h}{b} - 1\right)$$

$$\chi_{[11]} = \chi_{(11)} = \left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}\right)^{1/2}.$$

For more than three modes, this method of obtaining γ becomes too inefficient. For four or more modes, that is, in cases where a matrix of order 8 or higher is obtained, the propagation coefficient could be evaluated by finding the eigenvalues of the complex matrix.

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Letters

Transmission Line Transformation Between Arbitrary Impedances Using the Smith Chart

P. I. DAY

Abstract—**A graphical method for transforming between two complex impedances using a single transmission line matching section is described. The Smith chart is used in a mode where the chart normalizing impedance is arbitrary.**

In a recent letter, Arnold [1] presented a graphical method for transforming complex load impedances into resistive load impedances using a transmission line section, determining both the line impedance and length. The Smith chart was used although the line impedance was initially unknown. The method can be extended to cover the transformation between two complex impedances using a construction described in earlier literature for evaluating the line impedance in the complex transformation.

Somlo [2] showed that the characteristic impedance required for a line to transform two arbitrary impedances can be found by using an arbitrary normalizing impedance and constructing a circle centered on the real axis passing through the two impedance points. The technique was based on these properties of a Smith chart: 1) the locus of impedance along a loss-free line is always a circle; 2) the line impedance is given by the geometric mean of the circle intercepts with the real axis. A further property not mentioned by Somlo is that: 3) the square root of the ratio of the intercepts is the VSWR on the line.

The line length could be determined by reentering the Smith chart in the normal manner with the calculated line impedance. This letter

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suggests a method of determining the line length without reentering the chart and uses a construction similar to that proposed by Arnold [1].

We will consider the example of transforming $Z_L = (1 + j1)$ to $Z_S = (2 + j3)$. A suitable normalizing impedance is 2 Ω since this ensures that Somlo's circle through the two points is centered near the chart origin, the condition for greatest accuracy occuring when it is exactly centered. Somlo's construction yields a circle centered at C and with intercepts at 0.45 and 5.0; whence $Z_0 = 1.5$, this point is entered on the chart (Fig. 1). This construction has been left out for clarity but the chart center is marked; we note that this point only coincides with Z_0 either when both lie on the origin or the circle has zero radius. Construct a circle passing through Z_L and the ends of the real axis A and A'. Extend the line $Z_L - Z_0$ to the circle at P, then construct $P - 0 - l_L, l_L$ is then the angle equivalent of the reflection coefficient for Z_L. A similar construction for Z_S yields l_S and the length of transmission line required is the difference between the two values of l, the direction of rotation is as in normal Smith chart practice and $dl = 0.086\lambda$.

Use of the Smith chart in the arbitrary normalization mode enables one to define the range of impedances to which a known impedance may be transformed using a single matching section. Using $Z_L = (1 + j1)$ normalized to 2 Ω we enter the chart in Fig. 2. The impedance can be matched to any other provided the circle constructed through the two impedance points does not intercept the chart boundary at $\rho = 1$. This limits the range of impedances to which Z_L may be matched to the area shaded on Fig. 2.

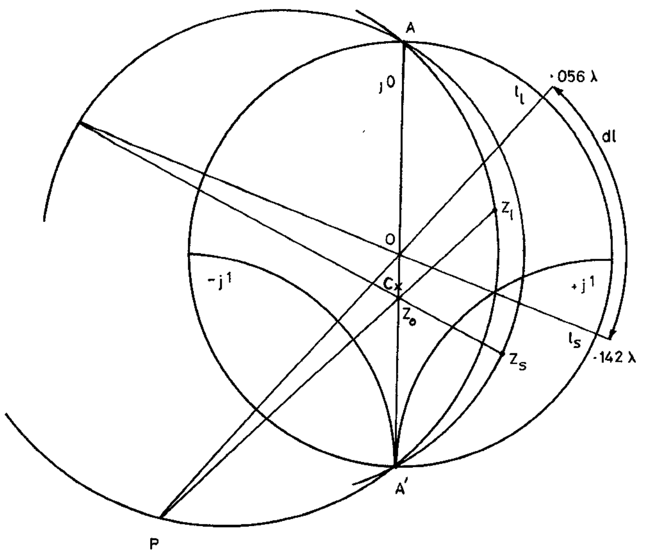

Fig. 1. Construction for transforming $Z_L = (1 + j1)$ to $Z_S = (2 + j3)$.

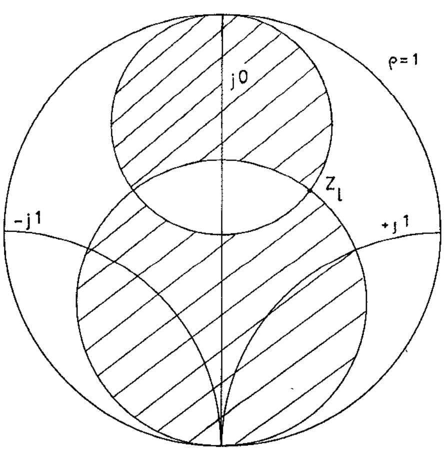

Fig. 2. Area of simple matching impedances to $Z_L = (1 + j1)$.

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Analysis of DC Blocks Using Coupled Lines

CHEN Y. HO

***Abstract*—It is shown mathematically that dc blocks can be realized by using λ/4 − 3-dB directional couplers with both coupled port and transmitted port open-circuited.**

Using λ/4 − coupled-line structure, dc blocks in microwave frequency have been realized in the past [1]. The approach of analysis used in [1] employs an approximate equivalent circuit based on the even- and odd-mode propagation in coupled lines of [2]. It is the purpose of this letter to show that this type of dc block can be analyzed exactly as a 3-dB directional coupler with both coupled port and transmitted port open-curcuited. The results are general and can be applied to any type of realization of 3-dB directional couplers, not necessarily restricted to microstrip edge-coupled lines.

The schemetic of conventional directional couplers is shown in Fig. 1 in which port 1 is assumed the input port, port 2 is the coupled port, port 3 is the isolated port, and port 4 is the transmitted port.

By assuming that ports 1 and 3 are terminated with the characteristic impedance Z_0, while ports 2 and 4 are open-circuited, the impedance matrix of this four-port network [3] can be simplified as follows:

$$\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{13} \\ Z_{31} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} \tag{1}$$

where V_1, I_1, V_3, and I_3 are voltages and currents at port 1 and port 3, respectively, and

$$Z_{11} = Z_{33} = \frac{Z_{0e} + Z_{0o}}{2s} = \frac{1}{s(1 - k^2)^{1/2}} Z_0 \tag{2}$$

$$Z_{13} = Z_{31} = \frac{(Z_{0e} - Z_{0o})\,(1 - s^2)^{1/2}}{2s} = \frac{k(1 - s^2)^{1/2}}{s(1 - k^2)^{1/2}} Z_0 \tag{3}$$

where

Z_{0e} even-mode impedance of the coupled lines;
Z_{0o} odd-mode impedance of the coupled lines, and $Z_{0e}Z_{0o} = Z_0^2$;
k voltage coupling coefficient;
s $(-1)^{1/2} \tan(\theta)$, θ is the electrical length.

After substituting the relation $V_3 = -Z_0 I_3$ into (1), and eliminating the variable I_3, we obtain

$$\frac{V_1}{I_1} = Z_{\text{in1}} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_0 + Z_{33}} \tag{4}$$

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